CHAPTER SIX

BINARY OPERATION

Operation rules in ordinary Algebra:

These operational rules are

1) **Closure:**

- A statement is opened when no limitation is placed on it, and closed when a limitation is placed on it.
- For example Kofi is a boy is an opened statement, but Kofi is a boy in the class is a closed statement.
- Also x is a number is an opened statement, but x is a number less than 10 is a closed one.

2) **Commutative law:**

a) If a + b = b + a, then the given operation which is + or addition is commutative.
b) If a × b = b × a, then the given operation which is × or multiplication is commutative.
c) Lastly if a Δ b = b Δ a, then the given operation which is Δ is commutative.

Q1) Using the numbers 3 and 4, determine whether or not addition is commutative.

N/B L.H.S = Left hand side and R.H.S= Right hand side.
Soln.

For addition (+) to be commutative, then 3+ 4 = 4 + 3.

Consider the L.H.S i.e 3 + 4 =7.

Consider the R.H.S i.e 4+3 =7.

Since L.H.S = R.H.S, then addition is commutative.

Q2) Using the numbers 3 and 4, determine whether or not subtraction is commutative.

Soln.

If subtraction is commutative, then 3 – 4 = 4 – 3.

Considering the L.H.S, 3 – 4 = -1.

Considering the R.H.S, 4 – 3 =1.

Since the L.H.S $\neq$ R.H.S

i.e L.H.S is not equal to the R.H.S, then subtraction is not commutative.

Q3) Using the numbers 3 and 4, determine whether or not multiplication is commutative.

Soln.

For multiplication to be commutative, then 3×4 = 4×3.

L.H.S = 3×4 =12.

R.H.S = 4×3 =12.

Since L.H.S = R.H.S, then the operation which multiplication is commutative.

3) Associative Law:

a) If (a + b) + c = a + (b + c), then addition is associative.
b) If (a × b) × c = a × (b × c), then multiplication is associative.
c) If (a * b) * c = a * (b * c), then the operation which is *, is associative.

Q1) Using the numbers 2, 3 and 5, determine whether or not addition is associative.

Soln.

If + (addition) is associative, then (2+3) +5 = 2+ (3+5).

L.H.S = (2+3) +5 = 5+5=10

R.H.S = 2+ (3+5) = 2+8=10

Since the L.H.S = R.H.S, then addition is associative.

Q2) Using 2, 3 and 5, determine whether or not multiplication is associative.

Soln.

If multiplication is associative, then $(2\times3) \times 5 = 2\times (3\times5)$.

L.H.S = $(2\times3) \times5 = 6\times5 = 30$

R.H.S = $2\times (3\times5) = 2\times15 = 30$

Since L.H.S = R.H.S => × (multiplication) is associative.

Q3) Using 2,3 and 5, determine whether or not subtraction is associative.

<u>Soln</u>

If subtraction (-) is associative, then $(5 - 3) – 2 = 5 - (3 - 2)$

L.H.S = $(5-3) –2 = 2 – 2 = 0$

R.H.S = $5 - (3 - 2) = 5 - 1= 4$

Since R.H.S ≠ L.H.S, then (–) or subtraction is not associative.

NB: $a\times (b + c)$ is the same as $a (b + c)$

4)<u>Distributive Law:</u>

- If $a (b + c)$ or $a \times (b + c) = ab + ac$, then multiplication is said to be distributive over addition (+), or multiplication is said to be distributive with respect to addition.

- Also if $a (b \Delta c) = ab \Delta ac$, then multiplication is distributive over Δ, or multiplication is distributive with respect to the operation Δ.

<u>An operation:</u>

* An operation is a symbol, with a given meaning.

* For example if a*b = 2a+b, then the symbol * becomes an operation, and a*b means that take twice of a and add it to b.

* Also given that $a \Delta b = a^2 + b^2$, then the symbol Δ becomes an operation, and a Δ b means add a squared to b squared.

* Other examples of operation which we are familiar with are addition (+), subtraction (-), division (÷) and multiplication (×).

* Lastly any symbol can be used to represent an operation, provided its meaning is given.

The Identity Element:

The identity element of a given operation has no effect on that given operation.

* For example the identity element of addition is 0 (zero), since any number added to zero gives us the same number. (i.e zero has no effect on addition).

* For examples are

3 + 0 = 3

5 + 0 = 5

2 + 0 =2

* The identity element of multiplication is one, since one has no effect on multiplication.

i.e 2×1=2

4×1= 4

5×1= 5

* Therefore assume Δ to be a given operation, and if i.e = the identity element of Δ, then

2 Δ i.e = 2

4 Δ i.e = 4

6 Δ i.e = 6

.

Q1)

Δ	1	2	3	4
1	4	1	7	2

2	6	2	3	3
3	5	3	4	5
4	1	4	1	4

The given table is that for the operation Δ. By making a careful study of it, determine the identity element of the given operation.

Soln.

From the table

1 Δ 2 = 1

2 Δ 2 = 2

3 Δ 2 = 3

4 Δ 2 = 4

=> Any number Δ 2 = that number

=> 2 has no effect on the given operation, and as such it is the identity element.

Q2)

*	2	3	5	7
2	3	1	2	4
3	1	4	3	2
5	7	3	5	6
7	6	2	7	4

The given table is that drawn for a certain operation, which is represented by the symbol *. By careful analysis, determine the identity element for the given operation.

Soln.

A careful study indicates the following:

2 * 5 = 2

3 * 5 = 3

5 * 5 = 5

7 * 5 = 7

- This implies that 5 had no effect on the given operation. Therefore the identity element = 5

Binary operation:

Q1) The binary operation $\bigtriangledown$ is defined on the set of natural numbers by $a \bigtriangledown b = a + b$.

Determine whether or not $\bigtriangledown$ is

i. commutative.
ii. associative.
iii. Is multiplication distributive, with respect to the given operation $\bigtriangledown$.

Soln.

1. For the operation $\bigtriangledown$ to be commutative, then

$a \bigtriangledown b = b \bigtriangledown a$

Consider L.H.S:

$a \bigtriangledown b = a + b$, => L.H.S = a + b

Consider the R.H.S. Since $a \bigtriangledown b = a + b$

=> $b \bigtriangledown a = b + a$, which can also be written as a + b.

=> R.H.S = a + b

Since the R.H.S = L.H.S, then the given operation is commutative.

ii) Let a, b and $c \in N$ i.e. be member of the set of natural numbers. Then for $\triangledown$ to be associative, $(a \triangledown b) \triangledown c = a \triangledown (b \triangledown c)$.

Consider the L.H.S

i.e. $(a \triangledown b) \triangledown c$, solve what is inside the bracket first.

i.e. $(a \triangledown b) = a + b$

=> $(a \triangledown b) \triangledown c = (a + b) \triangledown c$

But $a \triangledown b = a + b$

=> $(a + b) \triangledown c = (a + b) + c = a + b + c$

$\therefore$ L.H.S = a + b + c

Consider the R.H.S i.e.

$a \triangledown (b \triangledown c)$

Solve what is inside the bracket first => $(b \triangledown c) = b + c$

=> $a \triangledown (b \triangledown c) = a \triangledown (b + c)$

But $a \triangledown b = a + b$

=> $a \triangledown (b + c) = a + (b + c)$

= a + b + c => R.H.S = a + b +c

Since L.H.S = R.H.S, then $\triangledown$ is associative.

iii) For multiplication to be distributive with respect to $\triangledown$, then $a (b \triangledown c) = ab \triangledown ac$

Consider the L.H.S i.e. $a (b \triangledown c)$; solve what is inside the bracket first.

i.e. $(b \triangledown c) = b + c$

=> $a (b \triangledown c) = a (b + c) = ab + ac$

=> L.H.S = ab + ac

Consider the R.H.S

i.e. ab $\triangledown$ ac.

Since a $\triangledown$ b = a + b

=> ab $\triangledown$ ac = ab + ac

=> R.H.S = ab + ac

Since L.H.S = R.H.S, then multiplication is distributive with respect to $\triangledown$.